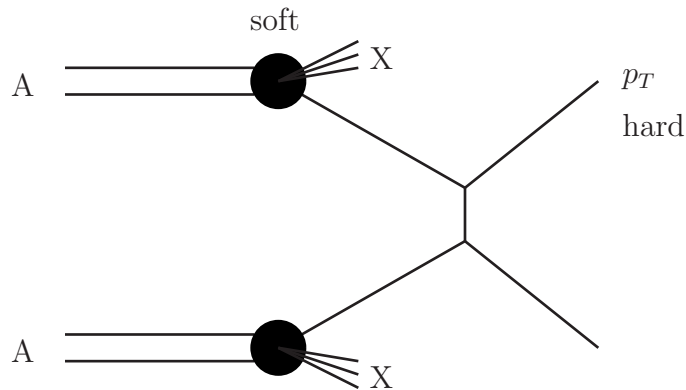# From QCD to heavy ion collisions

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This talk will discuss how heavy ion experiments, when moving from SPS $(10 + 10$ GeV) to RHIC (100+100 GeV) and to LHC (2750+2750 GeV), will enter a new domain of QCD in which the production of even large p_T gluons is so abundant that it is simultaneously perturbative but such that the phase space density of gluons is saturated. The saturation scale p_{sat} is estimated, quantitative numbers for the initial production of gluons at the LHC are given and options for their subsequent evolution are outlined. For parametrically large nuclei and energies, classical field methods will be applicable.

1. Perturbative calculability in heavy ion collisions

The first collisions at RHIC are expected to take place just during this PANIC conference and the outcome for, say, average charged multiplicity in a central collision will be known by the time the Proceedings are out. However, in spite of the fact that QCD is a perfectly well known theory we cannot with reliability predict this outcome – let alone the properties of QCD plasma produced in these collisions.
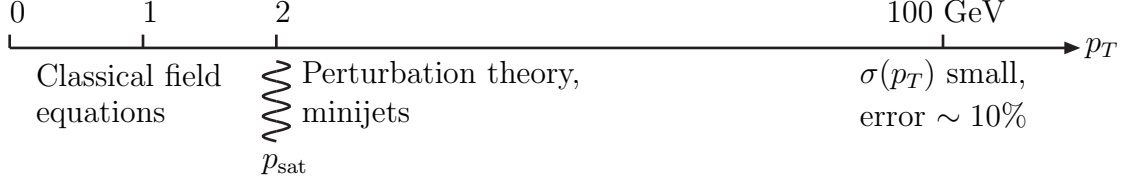
In perturbative QCD reliable predictions can be made for infrared safe quantities when there is a hard scale Q in the problem, A prototype is inclusive large p_T jet production, in which the hard (the parton-parton collision) and the soft (the structure functions at scale Q) parts of the problem can be factorised:

This schematic diagram is explicitly drawn for a nucleus-nucleus collision, though calorimetric jet measurements will need very large E_T due to the immense hadronic E_T back-

ground. Note also that the unobserved X's will in an average event contain a huge number of further $2 \to 2$ collisions.

Consider now the situation as a function of p_T (the scale is for LHC):

When one reduces the magnitude of p_T, the cross section grows and the error likewise. In this perturbative minijet picture one can compute the number of gluons with $p_T \geq p_0$ and the E_T they carry [1]-[4]. We can continue until p_0 reaches the saturation limit p_{sat}, to be defined below. Then the gluonic subsystem becomes very dense and new physics enters [5]-[6]. For $p_T \ll p_{\text{psat}}$ the phase space occupation number becomes $\gg 1$. Then this new physics may be effectively described by classical Yang-Mills equations [7], the initial conditions of which are given by an ensemble in colour space [8]-[9].

It is now crucial that for large nuclei and large energies the gluonic subsystem may become so dense that p_{sat} is in the perturbative region, $\gtrsim 2$ GeV.

Numbers for $p_T \gg p_{\text{sat}}$ can easily be produced using standard perturbative techniques. Classical field ideas are not yet sufficiently developed to permit the same for $\Lambda_{\text{QCD}} \ll p_T \ll p_{\text{sat}}$ and may ultimately require parametrically large nuclei ($A^{1/3} \gg 1$) and energies. To have some quidance from theory let us use the assumption already advocated in [5]: estimate the average event by taking $p_T = p_{\text{sat}}$. We are mainly interested in the local energy density or E_T (per unit rapidity): gluons with $p_T \gg p_{\text{sat}}$ carry lots of E_T but are very rare, whereas gluons with $p_T \ll p_{\text{sat}}$ are numerous but carry little E_T.

2. Minijet approach to the creation of early little bang

During the pre-little bang era we have two Lorentz contracted nuclei approaching each other. The system has zero temperature and zero entropy. The Lorentz contracted diameters, applicable to the valence quark parts of the wave functions, are

$$\frac{2R_A}{\gamma} = 1\,\text{fm} \qquad \text{SPS}, \tag{1}$$
$$= 0.1\,\text{fm} \qquad \text{RHIC},$$
$$= 0.005\,\text{fm} \quad \text{LHC}.$$

Already this naive fact emphasises the great qualitative difference between the energy domains. The little bang universe now is created by the nuclear collision, treated in the standard picture as a collision of two clouds of gluons (which dominate over quarks and antiquarks) with longitudinal momentum distributions $\approx Axg(x, p_T^2)$. The entropy content of this universe can be bounded from below by computing the number of gluons with $p_T > p_0 = 2$ GeV. The dominant longitudinal momentum fractions are (when gluons from the two nuclei have equal $p_L = p_T$)

$$x \sim \frac{2}{10} = 0.2 \qquad \text{SPS},$$

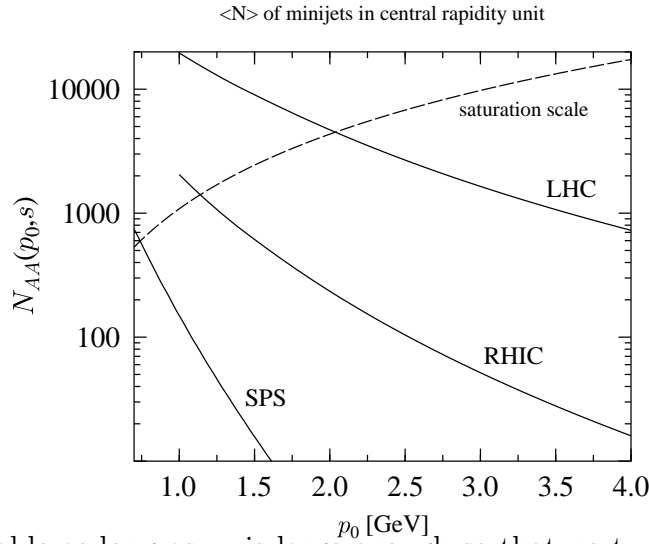$$\sim \quad \frac{2}{100} = 0.02 \quad \text{RHIC},$$

$$\sim \quad \frac{2}{2750} \approx 0.001 \quad \text{LHC}.$$

This is a second naive way of seeing the importance of increasing energy (keeping p_0 fixed): LHC can fully profit from the enhancement of gluons observed at small x [10].
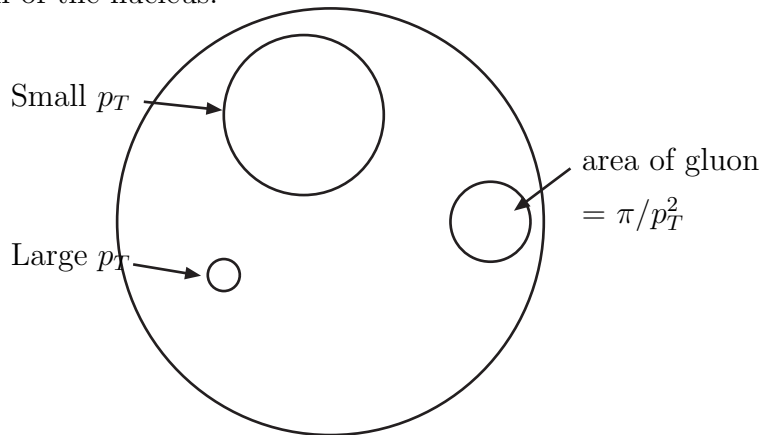
The computation of the number N_{AA} of gluons produced in a central A+A collision is quite straightworward [4]. What is important for a qualitative discussion is that this number scales $\sim A^{4/3}$. A hard cross section scales $\sim A^2$, but since one wants the number per inelastic collision, this has to be divided by $\pi R_A^2 \sim A^{2/3}$:

$$N_{AA} = \frac{A^2 \sigma(p_T > p_0)}{\pi R_A^2} \sim A^{4/3} \frac{1}{p_0^2}, \tag{2}$$

where σ is a $2 \to 2$ cross section. Computed values per unit rapidity at $y = 0$ are as follows (shadowing is included, $K = 1.5$):

<N> of minijets in central rapidity unit

The result is reliable as long as p_0 is large enough so that perturbation theory is valid and as long as the independent collision picture can be assumed to be valid. The latter becomes questionable when $p_0 < p_{\text{sat}}$, the saturation limit. To define p_{sat}, consider the transverse projection of the nucleus:

One can estimate the each gluon occupies the transverse area π/p_T^2 so that the gluon density saturates if

$$N_{AA} \times \pi/p_{\text{sat}}^2 > \pi R_A^2 \tag{3}$$

from which it follows that $p_{\text{sat}} \sim A^{1/6}$. This saturation limit is also shown in Fig.**??** and one sees that

$$
\begin{aligned}
p_{\text{sat}} &= 0.65\,\text{GeV} &\text{SPS}, \\
&= 1.1\,\text{GeV} &\text{RHIC}, \\
&= 2.0\,\text{GeV} &\text{LHC}.
\end{aligned}
\tag{4}
$$

Extending the computation to include partons with $p_T > p_{\text{sat}}$ one has, in one unit of rapidity:

SPS: 600 gluons with $p_T > 0.65$ GeV,

RHIC: 1100 gluons + 110 q + 80 $\bar{q}$ with $p_T > 1.1$ GeV,
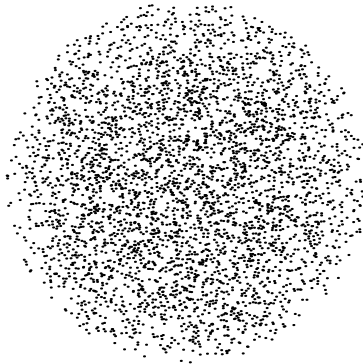
LHC: 4300 gluons + 200 q + 190 $\bar{q}$ with $p_T > 2.0$ GeV.

The corresponding E_T-values are

$$
\begin{aligned}
E_T &= 400\,\text{GeV} &\text{SPS}, \\
&= 2500\,\text{GeV} &\text{RHIC}, \\
&= 12000\,\text{GeV} &\text{LHC}.
\end{aligned}
\tag{5}
$$

Remember that these are the *initial* values at $\tau = \tau_i = 1/p_{\text{sat}}$.

It is, of course, questionable to extend the computation to the 1 GeV region, but the numbers are not unreasonable. However, due to the (relatively) small Lorentz contraction at SPS it is not clear how to convert to coordinate space there. At LHC the clocks can be started very accurately and one can say that at the LHC there will be at the time $\tau_i = 1/p_{\text{sat}} = 0.1$ fm after the collision about 4000 gluons with $p_T > p_{\text{sat}}$ in one unit of rapidity:

Geometrically, the volume of the little bang universe at the time of its creation is $V = \pi R_A^2 \star \tau_i \Delta y = \pi R_A^2/p_{\text{sat}} = 13\,\text{fm}^3$ for $A = $ Pb. This implies that the initial energy and number densities of gluons at the LHC are
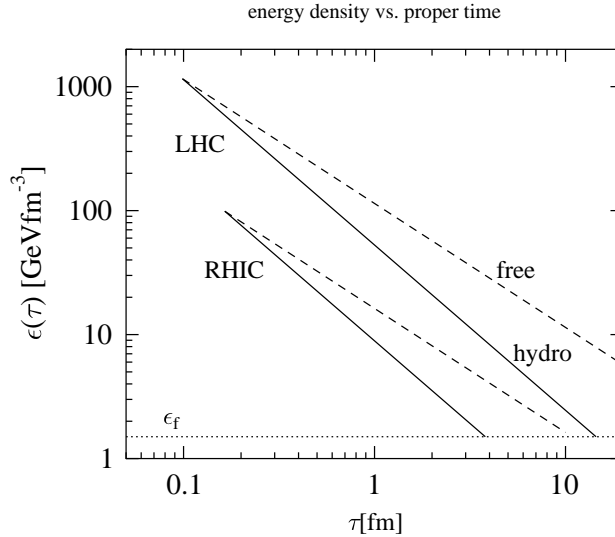
$$\epsilon_i = 970\frac{\text{GeV}}{\text{fm}^3}, \qquad n_i = \frac{320}{\text{fm}^3}. \tag{6}$$

3. Hubble expansion of the early little bang

Now comes the crucial question: what then? We have in usual QCD perturbation theory computed the composition of the $p_T > p_{sat} = 2$ GeV component of the system at the time $1/p_{sat} = 0.1$ fm. Invoking the assumption of [5], this is the initial state of an average event. But this is not yet something comparable with experiment. How does this subset behave for $\tau > 1/p_{sat}$ until conversion to hadrons? Of course, nobody knows definitely until a systematic non-perturbative computation in QCD has been carried out. Thus one has to make assumptions and these will be tested by experiment.

In this case it is quite natural to assume that the little bang universe is thermalised at creation at $\tau = 0.1$ fm (note that chemically (anti)quarks are not in equilibrium). Then it expands conserving the total entropy $S = 3.60N \approx 15000$. If we go to the center, the expansion is purely longitudinal, $S = sV \sim s\tau \sim T^3\tau = $ constant, $s \sim n \sim 1/\tau$ and $T \sim 1/\tau^{1/3}$ (compare $T \sim 1/t^{1/2}(\sim 1/t^{2/3})$ for the usual radiation (matter) dominated universe). However, due to $p\,dV$ work, energy density decreases faster [11]:

$$\epsilon \sim \frac{1}{\tau^{4/3}} \sim n\frac{1}{\tau^{1/3}}, \quad \frac{\epsilon}{n} \sim T \sim \frac{1}{\tau^{1/3}}. \tag{7}$$

energy density vs. proper time

Thus the initial and final entropies are the same. The final particles are dominantly pions, for which also $S \approx 4N$. Thus, simply, $N_{gluons} = N_{pions}$! E_T behaves quite differently:

$$E_{Tf} \lesssim \left(\frac{\tau_i}{\tau_f}\right)^{1/3} E_{Ti} \sim \frac{1}{6}E_{Ti}. \tag{8}$$

Thus the E_T goes down from 12000 GeV to about 2000 GeV. This plasma really does work! Stated in another way: the energy per particle went down from about 3 GeV to about 0.5 GeV. These predictions for Pb+Pb at LHC can easily be extended to any A and s.

Any dissipative effects will increase both the multiplicity and E_T. The extreme is free streaming: E_T constant. In this limit the multiplicity would be gigantic, about $E_T/0.5$GeV. If one could predict the initial parameters with a 10% accuracy, the measurement of final quantities would be a measurement of the degree of thermalisation. Or,
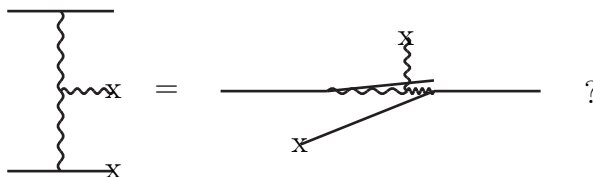
if one knew that the expansion is adiabatic, they would be a measurement of the initial conditions [12].

4. Classical fields

In the minijet computation above we basically used the $2 \to 2$ diagram (x denotes a large p_T external line):

but what about bremsstrahlung diagrams, like

This diagram predicts [13] a certain number of gluons around $y = 0$, but how to extend this computation to nuclear collisions? This is not as straightforward as in the minijet case: the p_T of the perturbative gluon is balanced by a "beam jet". The new ideas developed [8,9], [14]-[17] amount to computing classical radiation of gluons by two colour currents, formed by the valence quarks and the fastest gluons of the colliding nuclei, on the forward and backward light cones.

The motivation for the use of the classical field approximation is precisely that in the saturation limit the occupation number of any of the field modes is large and the classical field approximation should be appropriate. The situation is analoguous to that of computing the rate ("the sphaleron rate") of the baryon number violating reactions in electroweak matter [18]. These reactions also involve nonperturbative large field modes and their rate can only be computed numerically.

The essential parts of the classical field computation are the formulation of the equations of motion ("effective theory") and of the initial conditions. In the electroweak case, the standard model at finite T, the theory is weakly coupled and there is a perfectly controllable perturbative way [19] of computing the effective theory, which is a three-dimensional purely Euclidian SU(2)×U(1) gauge + fundamental (doublet) Higgs theory. The initial conditions are set by thermalising the system at some temperature, which is numerically straightforward. After that the classical field equations are integrated numerically in real time starting from some thermal field configuration and the sphaleron rate is computed as a real time correlation function of the topological susceptibility. Hereby one has learnt that the dynamics of the infrared small-momentum modes one is interested in is essentially affected by their interactions with the hard large-momentum ($\sim T$) modes. When this is correctly taken into account, the calculation of the sphaleron rate, which has no perturbative contribution, should be under control [20]. This is not yet the case with quantities like viscosities, which also have a perturbative contribution.

The situation is much more complicated in the application of the classical field approach to QCD in heavy ion collisions. Here there is no "top down" systematic way of deriving the effective theory, even in a theoretical weakly coupled limit and one has to proceed phenomenologically [8]. Let us focus our attention at the slice of matter at rest in the CMS. Then our space is again three-dimensional, but in this case Minkowskian with time and two spatial coordinates. The effective theory now is an SU(3) gauge + adjoint (octet) Higgs theory [17], but there is no known systematic way to derive its coefficients. To see why the theory is of this form, take the gluon part of the QCD action and include only longitudinal boost invariant configurations, independent of $\eta = \tanh^{-1}(z/t)$, which implies an infinite energy. The action then becomes

$$S[A_\mu^a, \phi^a] = \int d\tau\, d^2x \left[-\frac{\tau}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2\tau}(D_\mu \phi)^a (D^\mu \phi)^a \right],$$ (9)

where $\mu = 0, 1, 2$, A_1^a, A_2^a are the transverse components of the gluon field, ϕ^a is the component A_η^a of the gluon field and the gauge has been chosen as $A_\tau^a = 0$. This part of the action is singular at $\tau = 0$, on the light cone. This singularity is regulated by the dynamics of the nuclear interaction at $\tau = 0$, by the initial condition. One collision is represented by one random charge distribution $\rho_{(i)}^a(x_1, x_2)$ of the two colliding nuclei ($i = 1, 2$), where each of the two ρ's is drawn from a, say, Gaussian distribution of some width μ. The ρ's give the initial conditions for the fields $A_\mu^a(0, x_1, x_2), \phi^a(0, x_1, x_2)$ and the equations of motion give their further evolution in τ. Various physical quantities, like the number of bremsstrahlung gluons, can then be computed by averaging over many different evolutions corresponding to different initial conditions drawn from the Gaussian ensemble. A first attempt to carry out this program (for SU(2)) is in [17].

The crucial parameter is the average transverse colour charge density μ^2, which for the valence quarks of a nucleus is given by

$$\mu^2 = \frac{(N_c^2 - 1)}{2N_c^2} \frac{A}{\pi R_A^2},$$ (10)

but to which also the (dominating) contribution of the fast gluons has to be added [15]. The computation then is formally valid for

$$\Lambda_{\text{QCD}} \ll p_T \ll \mu,$$ (11)

where the lower limit expresses the fact that one is in the weakly coupled domain (α_s is parametrically small) and the upper that one simultaneously is in the saturation domain, occupation numbers are large. Parametrically, like p_{sat}, μ scales $\sim A^{1/6}$. However, numerically at LHC energies, $\mu \approx 0.8$ GeV [15], while p_{sat} was about 2.0 GeV. Converted to time units this means that at LHC the bremsstrahlung quanta thus will materialise at times $0.25\,\text{fm} \lesssim \tau \lesssim 1\,\text{fm}$, while the perturbative quanta materialised at $\tau \approx 0.1$fm. For practical applications, the window of applicability of the classical field approach thus seems very small, but conceptually it deals with an entirely new domain of QCD and thus studies with parametrically large $A, 1/g$ and $\sqrt{s}$ are very important.

However, even if one could solve the problem for all the scales $p_T \gg \Lambda_{\text{QCD}}$, implying the computation of the initial creation of the system at all times $\ll 1$ fm, there still remains the problem of all smaller p_T-scales, the entire further evolution of the system,

its expansion, conversion to the hadronic phase and ultimate decoupling. Here one in any case will have to resort to phenomenological ideas, which later will be checked by experiments.

5. Conclusions

Heavy ion experiments at RHIC and LHC will enter a new domain of QCD, in which the production of even large p_T gluons is so abundant that it can be simultaneously perturbative but such that that the phase space density of gluons is saturated. This talk has discussed precisely what numbers of gluons can be expected under these conditions to be initially (at 0.1 fm) produced at the LHC and what options there are for the subsequent behaviour of this system. Whatever the detailed validity of these numbers is, one can in any case with confidence look forward to abundant production of QCD plasma at RHIC and especially at the LHC.

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